JWA Fackrell - Bispectral Analysis of Speech Signals

# Notes on existing acoustic models

Assumptions made for general models: VT/source are uncoupled. Non-rigid VT walls, heat, conduction and viscous loss ignored. Acoustic waves propagate as plane waves.

Assumptions made for the linear model: Phase is not important. Excitation on the linear filter is Gaussian. The filter is linear.

# Evidence for nonlinearities in speech

Teager. Relationship between the air flow and pressure in the VT is not constant. Two types of techniques for nonlinear speech processing: (1) use nonlinear dynamics/chaos theory; (2) still treat the speech signals as a concatenation of quasi-stationary states, but use nonlinear models to describe these states.

# Higher order statistics

Explorative vs. exploitative techniques.

# Bispectral Analysis

Wide-sense stationary – arbitrary time shift to the signal does not change the joint moments up to order two. In other words, mean and covariance not changed.

Strict-sense stationary – all joint moments are not affected by time shift.

Definition of the moments:

* is the mean
* is the variance (spread of pdf)
* is the skewness (asymmetry of pdf)
* is the kurtosis (sharpness of peak)
* Moment generating function (MGF) = Taylor series with moments as the coefficients.

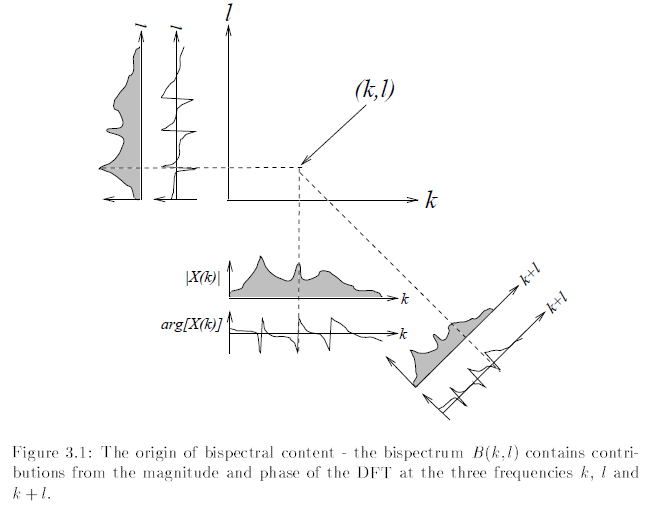
Definition of the cumulants:

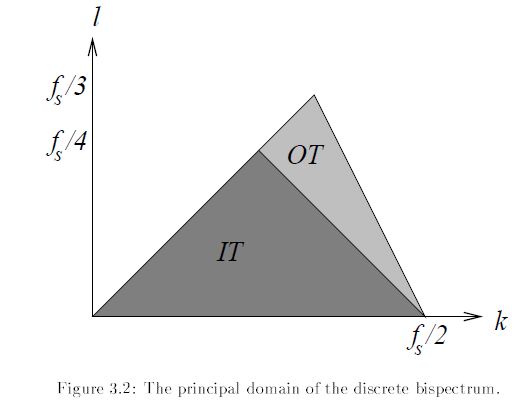
* Defined in terms of the Taylor expansion of the cumulant generating function (CGF)
* CGF = ln(MGF)
* For zero mean processes, and

Definition of polyspectra:

* Power spectral density .
* Bispectrum .
* Variance . Skewness

# The bispectrum

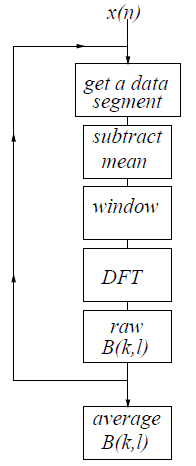




Power spectrum has a point of symmetry at the folding frequency . Bispectrum has same kind of symmetry. Principal domain is the region the contains no redundant information, which is a triangular region. (IT=inner triangle and OT=outer triangle).

# Bispectrum estimation

M1 = stochastic signal. M2 = mixture of deterministic (sinusoidal) and stochastic signals. Direct vs. indirect methods (periodogram averaging vs. cumulant estimation). Here direct method is used.



Signals conforming to M1:

* Smoothed bispectrum estimator is asymptotically complex normal (real and imaginary part are both normally distributed) and asymptotically independent (estimate at independent to ).
* Bispectral estimates are asymptotically unbiased (as data length increases, estimate tends towards true bisepctrum)
* . Suppose signal has more energy at than at , then the bispectral estimate will have higher variance at simply due to the energy difference.
* Since variance is energy dependent, bispectral estimate measures 2nd order and 3rd order properties. Need to try remove the second order sensitivity. One way is to pre-whiten the signal prior to bispectral analysis. More common way is to propose a new measure, called the skewness function
* Skewness function estimate would have flat variance if denominator was known exactly.

Signals conforming to M2:

* Do not conform to the statistical properties described above.
* Mean and variance of bispectral estimates depends on the relationships between frequencies of the component sinusoids.
* Noise free case is simplest.

Alternative normalization is

which can be estimated as

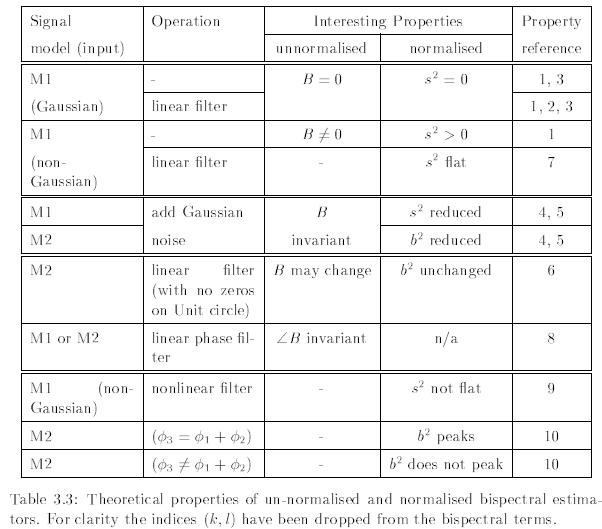
This is the normalised bispectrum, or squared bicoherence, which has the same approximately-flat variance that the skewness function has. Unlike the skewness function, it is bounded between zero and one.

Other normalizations are the Kravtechenko-Berejnoi estimate,

which has the advantage of the arguments of the denominator are symmetric under perturbations. Not used extensively. Another one is amplitude-only (AO) and phase-only (PO) measures,

# Properties of bispectrum

* Bispectrum of a Gaussian signal is identically zero
* Bispectrum of a linearly filtered Gaussian signal is identically zero
* Bispectrum of a non-Gaussian signal is blind to additive Gaussian noise
* Bicoherence of signal conforming to M1 or M2 is generally not blind to additive Gaussian noise
* If signal is filtered by a linear filter, then the magnitude of the normalised bispectrum is unchanged, provided that there are no zeros on the unit circle
* Skewness function of linearly filtered non-Gaussian iid signals is flat
* If signal is filtered by a linear-phase filter, the biphase information is unchanged
* Skewness function of a non-Gaussian M1 signal which has been passed through a nonlinear filter may not be flat.
* Bicoherence of a harmonic M2 signal peaks if the signal phases , and at frequencies , , respectively have the relation . (Quadratic phase coupling)



# Estimation Issues

* Issues regarding choice of window, comparison of theoretical and empirical results
* Data length issues. Bispectra have higher variances compared to PSD, so lengths that may be sufficient for PSD estimation might not be for bispectrum estimation
* Hinich et al. no. of segments should be at least as large as the DFT size (). For trispectrum estimation .
* Required length of data depends on how noisy the data is
* Theorem: If effects of leakage are ignored, then the peak bicoherence corresponding to frequencies , of an M2 signal consisting of three equal-amplitude coupled harmonics in variable levels of additive white Gaussian noise is

Peak bicoherence depends on the DFT size and SNR.

Bias and variance:

Replacing with the above equation gives formulas for the bias and variance of the estimate in terms of the SNR, DFT size and number of data segments .